ANALYSIS OF COLBURN'S ARITHMETICS. V

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Colburn's treatment of percentage, interest, etc., is perhaps typical of his attitude toward the applications of arithmetic. On p. 28 of the *Sequel*, in sec. VII, on multiplication, this paragraph is given immediately preceding the first problem on interest:

Interest is a reward allowed by a debtor to a creditor for the use of money. It is reckoned by the hundred, hence the rate is called so much per cent or per centum. *Per centum* is Latin, signifying by the hundred. 6 per cent signifies 6 dollars on a hundred dollars, 6 cents on a hundred cents, £6 on £100, etc., so 5 per cent signifies 5 dollars on 100 dollars, etc. Insurance, commission, and premiums of every kind are reckoned in this way. Discount is so much per cent to be taken out of the principal.

Colburn evidently considers this sufficient explanation for such problems as the following, for he gives nothing additional either here or in Part II.

44. What is the interest of $43.00 for 1 year at 6 per cent?

47. What is the interest of $247.00 for 3 years at 7 per cent?

49. I imported some books from England, for which I paid $150.00 there. The duties in Boston were 15 per cent, the freight $5.00. What did the books cost me?

53. A merchant bought a quantity of goods for 243 dollars, and sold them so as to gain 15 per cent; how much did he gain, and how much did he sell them for?

The next mention of percentage is on p. 77, sec. XXIV. This problem is given:

119. A merchant sold a quantity of goods for $273.00, by which he gained 10 per cent on the first cost. How much did they cost?

Following the problem, is this note:

Note.—10 per cent is 10 dollars on a 100 dollars, that is, $\frac{10}{100}$. 10 per cent of the first cost therefore is $\frac{10}{100}$ of the first cost. Consequently $273.00 must be $\frac{110}{100}$ of the first cost.

A little farther on in the list we find the following problems and notes:

122. A merchant sold a quantity of goods for $983.00, by which he lost 12 per cent. How much did the goods cost and how much did he lose?

Note.—If he lost 12 per cent, that is $\frac{12}{100}$, he must have sold for $\frac{88}{100}$ of what it cost him.

124. A merchant sold a quantity of goods for $87.00 more than he gave for them, by which he gained 13 per cent of the first cost. How much did the goods cost him, and how much did he sell them for?

Note.—Since 13 per cent is $\frac{13}{100}$, $87.00 must be $\frac{13}{100}$ of the first cost.

130. A man having put a sum of money at interest at 6 per cent, at the end of 1 year received 13 dollars for interest. What was the principal?

Note.—Since 6 per cent is $\frac{6}{100}$ of the whole, 13 dollars must be $\frac{6}{100}$ of the principal.

132. A man put a sum of money at interest for 1 year, at 6 per cent, and at the end of the year he received for the principal and interest 237 dollars. What was the principal?

Note.—Since 6 per cent is $\frac{6}{100}$, if this be added to the principal it will make $\frac{106}{100}$, therefore $237 must be $\frac{106}{100}$ of the principal. When interest is added to the principal the whole is called the *amount*.

133. What sum of money put at interest at 6 per cent will gain $53 in 2 years?

Note.—6 per cent for 1 year will be 12 per cent for 2 years, 3 per cent for 6 months, 1 per cent for 2 months, etc.

138. Suppose I owe a man, $287 to be paid in one year without interest, and I wish to pay it now; how much ought I to pay him, when the usual rate is 6 per cent?

Note.—It is evident that I ought to pay him such a sum, as put at interest for 1 year will amount to $287. The question therefore is like those above. This is sometimes called *discount*.

Later in the sections on decimal fractions, special methods for interest are given in the same way, i.e., by means of a note following a problem which calls for a special method.

EDUCATIONAL PRINCIPLES RECOGNIZED. THE INDUCTIVE METHOD

In the titles of both his arithmetics,[1] Colburn explicitly states that the method of presentation is inductive rather than deductive. We have already alluded to the instruction of Colburn's time as being a drill in the manipulation of written symbols as opposed to oral instruction which he introduced. This "old system" was also deductive. The rule was given to the pupil in the beginning and he was expected to interpret it and apply it to problems. In

[1] The full titles of the texts in the later editions are: *First Lessons, Intellectual Arithmetic upon the Inductive Method of Instruction*, and *Arithmetic upon the Inductive Method of Instruction: Being a Sequel to Intellectual Arithmetic.* These titles are taken from editions of 1847 and 1828 respectively. Even his algebra has the title, *An Introduction to Algebra upon the Inductive Method of Instruction.*

some of the better texts the rule was followed by two or three problems worked by the rule and explained. The presentation of division quoted from the *Scholar's Arithmetic* on p. 31 is a typical illustration of the deductive method.

The inductive method which Colburn presented is the reverse of this. The way in which he develops the topics of arithmetic may be illustrated by his development of division in the *Sequel.*

In Part I, p. 32, he begins division with these problems:

1. How many oranges, at 6 cents apiece, can you buy for 36 cents?
2. How many barrels of cider, at 3 dollars a barrel, can be bought for 27 dollars?
3. How many bushels of apples, at 4 shillings a bushel, can you buy for 56 shillings?
4. How many barrels of flour, at 7 dollars a barrel, can you buy for 98 dollars?
5. How many dollars are there in 96 shillings?

The table of English money is given following problem 5, and the list is continued with problems of this type. These problems are such that the pupil probably was able to solve the first ones intuitively. The list consists of 128 problems of gradually increasing difficulty. The last 22 only are abstract and they are intended simply for drill.

In Part II, p. 142, which is to be studied with Part I, we find:

A boy having 32 apples wished to divide them equally among 8 of his companions; How many must he give them apiece?

If the boy were not accustomed to calculating, he would probably divide them, by giving one to each of the boys, and then another, and so on. But to give them one apiece would take 8 apples, and one apiece again would take 8 more, and so on. The question then is, to see how many times 8 may be taken from 32; or, which is the same thing, to see how many times 8 is contained in 32. It is contained four times. *Ans.* 4 each.

A boy having 32 apples was able to give 8 to each of his companions. How many companions had he?

This question, though different from the other, we perceive, is to be performed exactly like it. That is, it is the question to see how many times 8 is contained in 32. We take away 8 for one boy, and then 8 for another, and so on.

A man having 54 cents, laid them all out for oranges, at 6 cents apiece. How many did he buy?

It is evident that as many times as 6 cents can be taken from 54 cents, so many oranges can he buy. *Ans.* 9 oranges.

A man bought 9 oranges for 54 cents; how much did he give apiece?

In this example we wish to divide the number 54 into 9 equal parts, in the same manner as in the first question we wish to divide 32 into 8 equal parts. Let us observe, that if the oranges had been only one cent apiece, nine of them would come to 9 cents; if they had been 2 cents apiece, they would come to twice nine cents; if they had been 3 cents apiece, they would come to 3 times 9 cents, and so on. Hence the question is to see how many times 9 is contained in 54. *Ans.* 6 cents apiece.

In all the above questions the purpose was to see how many times a small number is contained in a larger one, and they may be performed by subtraction. If we examine them again we shall find also, that the question was, in the two first, to see what number 8 must be multiplied by, in order to produce 32; and in the third, to see what the number 6 must be multiplied by, to produce 54; in the fourth, to see what number 9 must be multiplied by, or rather what number must be multiplied by 9, in order to produce 54.

The operation by which questions of this kind are performed is called *division.* In the last example, 54, which is the number to be divided, is called the *dividend;* 9, which is the number divided by, is called the *divisor;* and 6, which is the number of times 9 is contained in 54, is called the *quotient.*

Mr. Colburn then goes on to tell how to prove division and following this takes up the case when the combination is not one that has occurred in the multiplication table.

At 3 cents apiece, how many pears may be bought for 57 cents?

It is evident that as many pears may be bought, as there are 3 cents in 57 cents. But the solution of this question does not appear so easy as the last, on account of the greater number of times which the divisor is contained in the dividend. If we separate 57 into two parts it will appear more easy.

$$57=30+27$$

We know by the table of Pythagoras[1] that 3 is contained in 30 ten times, and in 27 nine times, consequently it is contained in 57 nineteen times, and the answer is 19 pears.

This same method is explained for four more problems in which he points out how the breaking up of the dividend may be determined. He then continues:

It is not always convenient to resolve the number into parts in this manner at first, but we may do it as we perform the operation.

In 126 days how many weeks?

[1] Multiplication table.

Operation

126 = 70 + 56 Instead of resolving it in this manner, we will write it down as follows:

$$\begin{array}{rrl} \text{Dividend} & 126 & (7 \text{ Divisor} \\ & \underline{70} & \underline{} \\ & 56 & 10 \\ & \underline{56} & \underline{8} \\ & & 18 \text{ quotient} \end{array}$$

I observe that 7 cannot be contained 100 times in 126, I therefore call the two first figures on the left 12 tens or 120, rejecting the 6 for the present. 7 is contained more than once and not so much as twice in 12, consequently in 12 tens it is contained more than 10 times and less than 20 times. I take 10 times 7 or 70 out of 126, and there remains 56. Then 7 is contained 8 times in 56, and 18 times in 126. *Ans.* 18 weeks.

From this the development is continued through four more problems, the last only being abstract and having a divisor of five digits. The rule is then stated, the last thing in the section.

MOTIVATION

The development of a motive by means of problems which especially appeal to children and by causing the child to feel a need for the process or definition before it is given to him is an important feature of both texts. The types of problems are well illustrated by those already given.

A feeling of need for the process is created by introducing each topic by problems. The very plan of dividing the texts into two parts and thus separating the problems from the development of the principles operates to create motive for the study of the principles. Even in the development of the principles, the rules are not stated until after the explanation of the operation which is itself based upon a problem. Whatever drill seems necessary is not given until after a considerable number of problems have been solved by the pupil. Colburn clearly states his attitude toward rules. He says:

> To succeed in this (i.e., teaching arithmetic to children), however, it is more necessary to furnish occasions for them to exercise their own skill in performing examples, rather than to give them rules.

But even these devices do not represent all that Colburn has done to motivate the arithmetic work. His style of writing and

his ability to see things from the child's point of view assist materially in this respect. But the way he guides the learner in the development of the principles adds a touch of genius to the whole work. The following is from the *Sequel*, p. 168:

A boy wishes to divide $\frac{3}{4}$ of an orange equally between two other boys; how much must he give them apiece?

If he had 3 oranges to divide, he might give them 1 apiece, and then divide the other into two equal parts, and give one part to each, and each would have $1\frac{1}{2}$ orange. Or he might cut them all into two equal parts each, which would make six parts, and give 3 parts to each, that is, $\frac{3}{2} = 1\frac{1}{2}$, as before. But according to the question, he has $\frac{3}{4}$ or 3 pieces, consequently he may give 1 piece to each, and then cut the other into two equal parts, and give 1 part to each, then each will have $\frac{1}{4}$ and $\frac{1}{2}$ of $\frac{1}{4}$. But if a thing be cut into four equal parts and then each part into two equal parts, the whole will be cut into 8 equal parts or eighths; consequently $\frac{1}{2}$ of $\frac{1}{4}$ is $\frac{1}{8}$. Each will have $\frac{1}{4}$ and $\frac{1}{8}$ of an orange. Or he may cut each of the three parts into two equal parts, and give $\frac{1}{2}$ of each part to each boy, and then each will have 3 parts, that is $\frac{3}{8}$. Therefore $\frac{1}{2}$ of $\frac{3}{4}$ is $\frac{3}{8}$. *Ans.* $\frac{3}{8}$.

Two more problems are similarly explained, though somewhat more briefly. He then draws a conclusion as follows:

In the last three problems the division is performed by multiplying the denominator. In general, if the denominator of a fraction be multiplied by 2, the unit will be divided into twice as many parts, consequently the parts will be only one half as large as before, and if the same number of the small parts be taken, as was taken of the large, the value of the fraction will be one half as much. If the denominator be multiplied by three, each part will be divided into three parts, and the same number of parts be taken, the fraction will be one third of the value of the first. Finally if the denominator be multiplied by any number, the parts will be so many times smaller. Therefore, *to divide a fraction, if the numerator cannot be divided exactly by the divisor, multiply the denominator by the divisor.*

In the above illustration and also the development of division which we have already quoted, Colburn takes as his point of departure a crude, intuitional way of handling the problem. This way is one which the pupil would probably discover for himself. It is only after he has presumably found this way rather cumbersome that a shorter way and rule are given to him. On just this point Colburn makes clear his attitude in the preface of the *Sequel:*

When the pupil is to learn the use of figures for the first time, it is best to explain to him the nature of them as in Art. I, to about three or four places; and

then require him to write some numbers. Then give him some of the first examples as in Art. II, without telling him what to do. He will discover what is to be done, and invent a way to do it. Let him perform several in his own way, and then suggest some method a little different from his, and nearer the common method. If he readily comprehends it, he will be pleased with it, and adopt it. If he does not, his mind is not yet prepared for it, and should be allowed to continue his own way longer, and then it should be suggested again. After he is familiar with that, suggest another method, somewhat nearer the common method, and so on, until he learns the best method. Never urge him to adopt any method until he understands it, and is pleased with it. In some of the articles, it may perhaps be necessary for young pupils to perform more examples than are given in the book.

When the pupil is to commence multiplication, give him one of the first examples in Art. III, as if it were an example in Addition. He will write it down as such. But if he is familiar with the "First Lessons," he will probably perform it as multiplication without knowing it. When he does this, suggest to him, that he need not write the number but once. Afterwards recommend to him to write a number to show how many times he repeated it, lest he should forget it. Then tell him it is Multiplication. Proceed in a similar manner with the other rules.

One general maxim to be observed with pupils of every age is never to tell them directly how to perform any example. If a pupil is unable to perform an example, it is generally because he does not fully comprehend the object of it. The object should be explained, and some questions asked, which will have a tendency to recall the principles necessary. If this does not succeed, his mind is not prepared for it, and he must be required to examine it more by himself, and to review some of the principles which it involves. It is useless for him to perform it before his mind is prepared for it. After he has been told, he is satisfied, and will not be willing to examine the principle, and he will be no better prepared for another case of the same kind, than he was before. When the pupil knows that he is not to be told, he learns to depend upon himself; and when he once contracts the habit of understanding what he does, he will not easily be prevailed on to do anything which he does not understand.

Several considerations induce the author to think, that when a principle is to be taught, practical questions should first be proposed, care being taken to select such as will show the combination in the simplest manner, and that the numbers be so small that the operation shall not be difficult. When a proper idea is formed of the nature and use of the combination, the method of solving these questions with large numbers should be attended to. This method, on trial, has succeeded beyond his expectations. Practical examples not only show at once the object to be accomplished, but they greatly assist the imagination in unfolding the principle and discovering the operations requisite for the solution.

Oral instruction and the *inductive method* are the features of Colburn's books which have received general recognition, and which were most effective in changing school practices. These two features were the essential components of the "new system" of teaching arithmetic to which Colburn makes reference in his address on the "Teaching of Arithmetic." Other features of his books have not been appreciated except by few. This is true of his concept of the subject-matter of arithmetic and its structure, the omission of certain topics, creating a motive for a process before it is presented, minimizing abstract drill and putting it after the concrete problems. Some of these things have been recognized within the past few years and others are now only beginning to be appreciated.

But, notwithstanding this failure to appreciate certain features, sufficient recognition was accorded oral instruction and the inductive method to change within a very few years both the actual teaching of arithmetic and the contemporary texts. The *Scholar's Arithmetic* by Daniel Adams has been cited as an example of the old type of text. In 1827 Mr. Adams published *Adams' New Arithmetic*. In the preface he says:

> The *Scholar's Arithmetic*, published in 1801, is synthetic. If that is a *fault* of the work, it is a fault of the *times* in which it appeared. The analytic or inductive method of teaching, as now applied to elementary instruction, is among the improvements of later years. Its introduction is ascribed to Pestalozzi, a distinguished teacher in Switzerland. It has been applied to arithmetic, with great ingenuity, by Mr. Colburn, in our own country.
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> The analytic is unquestionably the best method of *acquiring* knowledge; the synthetic is the best method of *recapitulating*, or *reviewing* it. In a treatise designed for school education, *both* methods are useful. Such is the plan of the present undertaking, which the author, occupied as he is with other objects and pursuits, would willingly have foreborne, but that, the demand for the *Scholar's Arithmetic* still continuing, an obligation, incurred by long-continued and extended patronage, did not allow him to decline the labour of a revisal, which should adapt it to the present more enlightened views of teaching this science in our schools. In doing this, however, it has been necessary to make a new work.

Speaking of the *First Lessons*, Mr. Page said in 1843 in addressing the American Institute of Instruction:

The reason, the understanding, is addressed, and led on step by step, till the whole is taken into the mind and becomes a part of it. The memory is little thought of, yet the memory cannot let it slip; for what has been drunk in, as it were, by the understanding, and made a part of the mind, the mind never forgets! To how many a way-worn and weary pupil under the old system; to how many a proficient, who could number his half dozen authors, and twice that number of manuscript cyphering books; to how many a *teacher* even, who had taught the old system, winter after winter, and yet saw but as "through a glass darkly"; to how many such, was this book on its appearance *Their First Lessons in Arithmetic?* WARREN COLBURN'S name should be written in a conspicuous place, in letters of gold, for this service.